# Engineering Notes

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## Lift and Pitching Moment Coefficient Changes from Low-Level Freestream Turbulence

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### Nomenclature

| | |
|---|---|
| $A$ | = speed of sound |
| $C_p$ | = local pressure coefficient |
| $M$ | = Mach number |
| $Q$ | = divergence of the velocity potential |
| $u'$ | = velocity perturbation |
| $(\overline{u'^2})^{1/2}$ | = rms of velocity perturbations |
| $U$ | = local velocity |
| $\gamma$ | = ratio of specific heats of fluid |
| $\phi$ | = instantaneous velocity potential |
| $\phi_o$ | = mean velocity potential |
| $\phi'$ | = velocity potential perturbation |
| $(\overline{\phi'^2})^{1/2}$ | = rms of velocity potential perturbations |
| $\Omega$ | = distant acting or body force per unit mass |
| $(\ )_\infty$ | = freestream conditions |

### Introduction

THE effects of grid-generated turbulent flow past airfoils have received a small amount of attention. Goldstein[1] reported on experimental data obtained in a wind tunnel with vertical steel strips located at the inlet of the test section. However, this arrangement obviously produced turbulence that was nonisotropic due to the orientation of the strips. Also, Payne and Nelson,[2] while only intending to study the effects of shear flow on an airfoil, found that the increase in turbulence due to the presence of the shear-producing screen had a definite effect on their results. They reported that there was a fivefold increase in the freestream turbulence intensity when the shear screen was in place. They concluded that it was this increase in turbulence—not the shear flow—that resulted in an increase in lift at a given angle of attack (as long as a separation bubble was present). They based their conclusions on the work done by Gault.[3] Gault found that increasing the turbulence level in the freestream effectively increased the Reynolds number of the flow around the airfoil model in the sense that the boundary-layer transition point moved closer to the leading edge. As a result there was an increase in the size of the turbulent boundary layer and a reduction of the amount of laminar separation on the suction side of the airfoil. This accounted for the increase in lift. Though the turbulence produced by Payne and Nelson[3] was again nonisotropic, the results are significant for comparison purposes. It is also important to note an interesting result obtained by Paterson and Amiet.[4] Their efforts centered on investigating the noise produced by an airfoil that was immersed in a turbulent flowfield that did approach the conditions of homogeneity and isotropy. They found that the chordwise pressure distribution on the airfoil in a turbulent flow tended to peak closer to the leading edge than when the incoming flow was smooth.

The purpose of this Note is twofold. First, the changes in the behavior of the lift and pitching moment coefficients for two different airfoils as a function of the freestream turbulent intensity will be experimentally obtained. Second, a prediction model for the lift coefficient variation will be developed and compared to the experimental data.

### Experimental Design

The NACA-0012 and NASA LS(1)-0417 airfoils each have a chord length of 20.5 cm and a total span of 45 cm. The airfoils are mounted vertically in the test section of the open-return-type wind tunnel. The test section dimensions are 62 cm in width by 45 cm in height and 244 cm in length. A 60-hp motor with variable-pitch fan blades provides flow speeds in the test section from 5–80 m/s.

For this investigation, the tunnel speed is initially set at 10.7 m/s, with the resulting Reynolds number based on a chord length of 145,000. The turbulent intensities in the tunnel flow are approximately 0.6% in the test section. In order to generate a quasi-isotropic turbulent flow at higher turbulence levels, a rectangular grid is installed immediately upstream of the test section. The grid consists of rods such that the mesh area is equal to 2.25 cm$^2$. The porosity of the grid is 70% and the Reynolds number based on the mesh is 15,500. The airfoil's leading edge is positioned 71 mesh diagonal lengths downstream. With the grid in the tunnel, the speed is increased to 11.4 m/s, with the gross Reynolds number increased to 154,000. At the location of the test models, the freestream turbulence intensity is 1.8%, which represents a threefold increase over the grid-free case. The effects of the Reynolds number are determined by repeating the investigation at a tunnel speed equal to 37 m/s ($Re = 500{,}000$) for the grid-out cases and 35 m/s ($Re = 475{,}000$) for the grid-in cases.

A single-component, helium-neon, frequency tracker-based laser-Doppler velocimeter in the backward scatter mode of operation is used to measure the velocity fields around each of the airfoils for both the grid-free and grid-included cases. A constant temperature hot-wire anemometer is used to supplement the laser velocimetry data close to the surface of the airfoils. Olive oil is used as the source of light scattering, with the average particle diameter equal to 1 $\mu$m. The static pressures are calculated from the velocity fields around the airfoils. Assuming constant pressure across the boundary layers, the static pressure immediately outside the boundary layer is set equal to the surface pressure at that location. With the surface pressure distributions, the lift and pitching moment coefficients are calculated at angles of attack ranging from 0 deg to near stall.

Corrections suggested by Rae and Pope[5] are applied to the lift and pitching moment data to account for the effects of solid blockage and streamline curvature caused by the models. Corrections generally decrease the lift and pitching

Received Feb. 26, 1987; revision received March 23, 1987.

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moments by 1-4%. Wake blockage corrections are not made because of the lack of drag data.

## Predictive Model

Consider an isentropic flow with a constant specific heat ratio $\gamma$. The formula for the local pressure coefficient is

$$C_p = \frac{2}{\gamma M^2}$$

$$\left\{\left[1-\frac{\gamma-1}{A_\infty^2}\left(\frac{\partial\phi}{\partial t}+\frac{Q^2-U_\infty^2}{2}+\Omega_\infty-\Omega\right)\right]^{\gamma/(\gamma-1)}-1\right\} \quad (1)$$

The argument is made that for two-dimensional wings, it is acceptable to superimpose both the thickness and the lifting contributions of the steady field but that neither has any first-order influence on the unsteady loading.[6] Using a perturbation analysis yields

$$C_p = -2\frac{\partial\phi}{\partial x} - \frac{2}{U_\infty}\frac{\partial\phi}{\partial t} + 0(\epsilon^2) \quad (2)$$

Assuming that Taylor's hypothesis is applicable, then to first order:

$$C_p = -4\frac{\partial\phi}{\partial x} \quad (3)$$

Next, consider the velocity potential for a slightly turbulent freestream:

$$\phi = \phi_o + \phi' \quad (4)$$

Then, the time-averaged pressure coefficient becomes

$$\overline{C_p} = -4\frac{\partial}{\partial x}(\overline{\phi_o\phi'}) \quad (5)$$

$$= -4\frac{\partial}{\partial x}\phi_o - 4\frac{\partial}{\partial x}(\overline{\phi'^2})^{1/2} \quad (6)$$

For relatively low freestream turbulent intensities, the change in the mean pressure coefficient as a function of freestream turbulence is

$$\overline{\Delta Cp} \cong -4\frac{\partial}{\partial x}(\overline{\Delta\phi'^2})^{1/2}$$

or

$$\overline{\Delta Cp} \cong -4\Delta\left[\frac{(\overline{u'^2})^{1/2}}{U}\right] \quad (7)$$

In the reported investigation, the freestream turbulent intensities are increased threefold from approximately 0.6 to 1.8%. Thus, according to Eq. (7), this increase should result in an approximately 12% decrease in the mean pressure coefficient. For a given wing section, the resultant decrease in the lift coefficient is of a comparable amount.

## Experimental Results

In Fig. 1, the lift and pitching moment coefficients for flow past the NACA-0012 are shown as a function of the angle of attack for the grid-out $\{[(\overline{u^2})^{1/2}/U] \sim 0.6\%\}$ and the grid-in $\{[(\overline{u^2})^{1/2}/U] \sim 1.8\%\}$ cases. In addition, data from both the present investigation at a lower Reynolds number ($Re = 1.5\times10^5$) and those obtained from previous work[7] are included. Several observations can be made. First, as the Reynolds number is increased, the results for the grid-out portion of the present investigations indicate an increase in the lift curve slope as expected, asymptotically approaching the ideal flow solution. Second, the presence of the turbulent generating grid results in a decrease in the magnitude of the lift coefficient at each angle of attack before separation and a delay in the attainment of $C_{\ell max}$ (i.e., $C_{\ell max}$ occurs at a larger angle of attack). The difference between the lift coefficient for the two levels of freestream turbulence is approximately 15%. Last, note that the pitching moment coefficient remains positive throughout for the grid in case. This seems to be another indication of the delaying of the stall for the

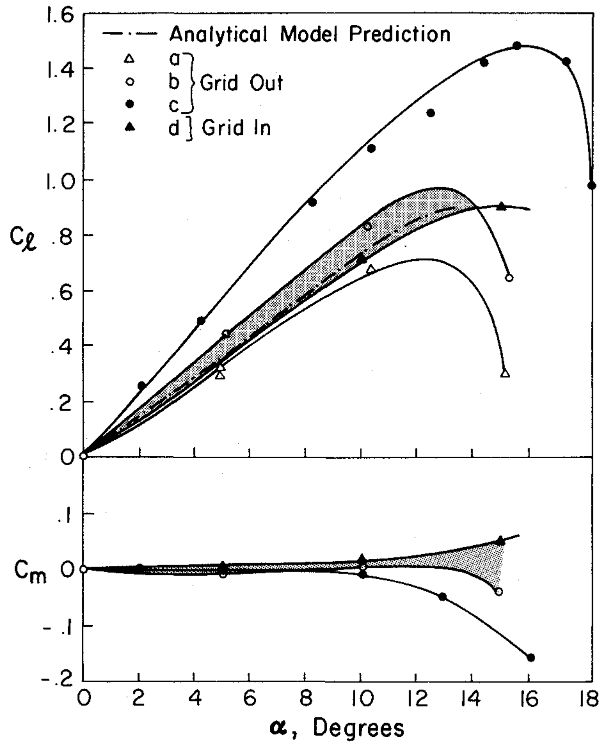


**Fig. 1 Comparison of grid-out vs grid-in cases for NACA-0012: a, $Re = 1.5\times10^5$; b, $Re = 5\times10^5$; c, $Re = 3\times16^6$ (from Ref. 7); and d, $Re = 4.75\times10^5$ (grid in.).**

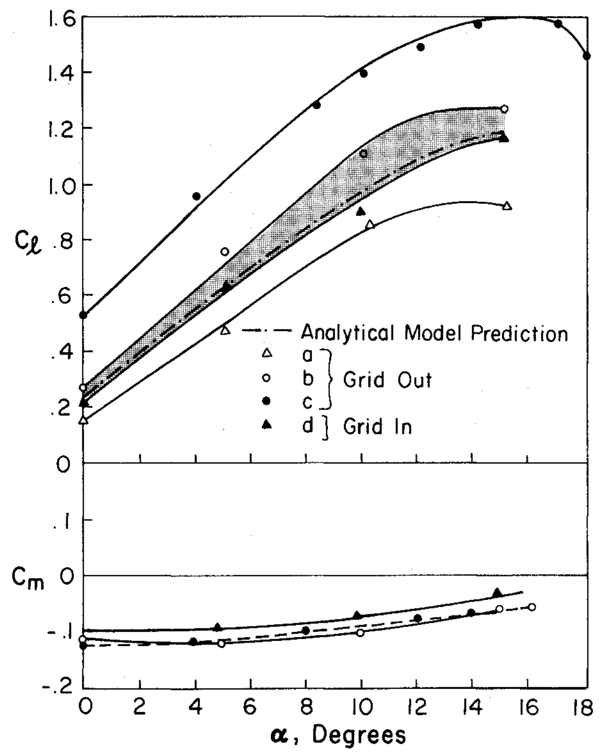


**Fig. 2 Comparison of grid-out vs grid-in cases for NASA LS(1)-0417: a, $Re = 1.5\times10^5$; b, $Re = 5\times10^5$; c, $Re = 1.9\times10^6$ (from Ref. 8); and d, $Re = 4.75\times10^5$ (grid in.).**

airfoil. A comparison between the experimental data for $C_\ell$ with and without the grid in place reinforces the credibility of the prediction model prior to stall.

In Fig. 2, similar comparisons and observations can be made for the flow past a NASA LS(1)-0417. First, note the trend of increasing $C_\ell$ as the Reynolds number is increased.[8] Second, the experimental data for $C_\ell$ show that with the grid in place there is again a significant decrease of approximately 16%. This compares favorably to the analytical model's predictions of a net decrease of 12%. With respect to the pitching moment coefficient, the turbulence-generating grid in place results in an approximately 20% reduction in the absolute value at each angle of attack.

## Conclusions

An increase in the freestream turbulent intensity from 0.6 to 1.8% results in a 15% decrease in the lift coefficient for the NACA-0012 and for the NASA LS (1)-0417. A first-order approximation for the local pressure coefficient suggests that the reduction in the lift coefficient would compare to approximately four times the change in turbulent intensity, or 12%. Thus, the predicted results compare quite favorably with the experimental evidence.

## References

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# Solution of Aerodynamic Integral Equations Without Matrix Inversion

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## Introduction

THE formulation of aerodynamic problems often leads to the solution of a linear integral equation of a singular type. Special care is needed to obtain a satisfactory solution, particularly when the geometry is complex. The geometry of aerodynamic shapes is usually defined by a certain number of node points. Most of the known numerical methods produce a matrix equation that can be solved using any of the standard routines such as Gaussian elimination.[1] However, the matrix involved, often called a matrix of influence coefficients, is full and can be quite large. A large matrix of influence coefficients can become ill-conditioned for some applications. Also, the solution is sensitive to the node point distribution; a poor distribution will almost always fail to give reasonable results.

In what follows, a new efficient method is presented that permits control over the accuracy of the solution and is less sensitive to the node point distribution. This method has the advantage of avoiding matrix inversion. Instead, simple scalar products are utilized. Finally, this method can be implemented easily on a microcomputer and even on a programmable pocket calculator. The approach is illustrated by calculating the pressure distribution over a Joukowsky airfoil. However, this method is quite general and can have a wide range of applications.

## Analysis

The calculation of the potential flow around an airfoil can be found by solving an integral equation of the following type[2,3]:

$$\int K(s,t)\gamma(s)\mathrm{d}s = f(t) \tag{1}$$

together with the Kutta-Joukowsky condition of zero vortex strength at the trailing edge. Making use of the continuity of the vortex density function, a smooth solution, more accurate for a higher number of node points, can be constructed. Consider a set of $m$ orthogonal functions, say

$$\{\phi_k\} \qquad k=1,m$$

By substituting each of these orthogonal functions to the vortex density function into the left-hand side of Eq. (1), a new set of functions $\{g_k\}$ is obtained, defined as

$$g_k(t) = \int K(s,t)\phi_k(s)\mathrm{d}s \tag{2}$$

The functions $f$ and $g_k$, for all $k$, can be specialized to $N$ control points around the airfoil. Thus, we construct a set of vectors

$$F_0 \text{ and } \{G_k\} \qquad k=1,m$$

The initial vector $F_0$ is an element of an Euclidian space of dimension $N$; the interest is to find the closest possible approximation of $F_0$ within the subspace of dimension $m$ spanned by the vectors $G_k$.

A sequence of vectors $F_k$ given by the following recurrence relation:

$$F_k = F_{k-1} - \lambda_k G_k \tag{3}$$

are generated in such a way as to minimize the distance $|F_k|$. The shortest distance is simply given by the orthogonal projection of the extremity of vector $F_k$ into the direction of vector $G_k$.

The scalar $\lambda_k$ is then uniquely determined as

$$\lambda_k = F_{k-1}\cdot G_k / G_k \cdot G_k \tag{4}$$

The residual vector $F_m$ obtained after exhausting the whole set of vectors $\{G_k\}$ can be used to introduce a quality factor, namely

$$q = |F_m| / |F_0| \tag{5}$$

If this quality factor is not significantly smaller than unity, the process we just described can be applied iteratively to give a better approximation for the residual vector and so on. In such a case, an overall quality factor can be easily defined as the product of individual quality factors for each iteration,

$$Q = q^1 \times q^2 \times q^3 \ldots \tag{6}$$

Received July 30, 1986; revision received Oct. 20, 1986. 

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